Int J Theor Phys (2008) 47: 1699–1708
DOI 10.1007/s10773-007-9611-z

Femtosecond Pulse Propagation in Optical Fibers Under Higher Order Effects: A Collective Variable Approach

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Received: 29 August 2007 / Accepted: 31 October 2007 / Published online: 14 November 2007

Abstract Employing collective variable approach, femtosecond pulse propagation has been investigated in optical fibers using the higher order nonlinear Schrödinger equation. In order to view the pulse dynamics along the propagation distance, variation of different pulse parameters, called collective variables, such as pulse amplitude, width, chirp, pulse center and frequency has been investigated by numerically solving the set of ordinary equations obtained from collective variable approach.

Keywords Optical solitons · Collective variables · Soliton perturbation · Optical fibers

1 Introduction

The propagation of optical solitons through optical fibers was first suggested by Hasegawa and Tappert [1], and first experimentally realized by Mollenauer et al. [2]. Since then, the study of optical soliton propagation through fibers and guided structures have gained tremendous attention owing to its fundamental aspects and as well as for important applications [1–9]. Propagation of femtosecond pulse in fiber links can be described by the nonlinear Schrödinger equation with higher order nonlinear terms [1–9]. Radhakrishnan, Kundu and Lakshmanan (RKL) proposed such a higher order nonlinear Schrödinger equation (HONLSE) [10], which takes care of the second as well as third order dispersion effects, cubic and quintic self phase modulations, self steepening and nonlinear dispersion effects. The normalized RKL model of HONLSE for the propagation of femtosecond pulse can be written as

$$i\psi_z + \psi_{tt} + 2|\psi|^2\psi + ic_1\psi_{ttt} + ic_2(|\psi|^2\psi)_t + ic_3(|\psi|^4\psi)_t + \alpha|\psi|^4\psi = 0, \qquad (1)$$

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where $\psi(z,t)$ represents the normalized complex slowly varying amplitude of the soliton field along the normalized distance z through the fiber, t is the normalized time with reference to a frame moving along the fiber with group velocity of the pulse. ψ_{tt} represents the normalized group velocity dispersion, with the next cubic term describing self phase modulations due to Kerr effect. The term involving c_1 represents the third order dispersion, while the term involving c_2 is the self steepening parameter. The sixth and seventh terms in the left hand side respectively describe the nonlinear dispersion and self phase modulation effects due to quintic nonlinearity. For pulse width greater than 100 fs, all higher order terms containing c_1, c_2 and c_3 are negligible and the HONLSE reduces to the NLSE with cubic quintic nonlinearity.

The HONLSE of the RKL model is not completely integrable. The nonintegrability arises due to the presence of higher order nonlinear and higher order dispersion terms. Both the analytical as well as numerical solutions of the HONLSE have been subject of intensive investigations. Various methods have been employed for the analytical study of soliton solution of the NLSE. The Lagrangian variational method [11, 12] being the most common of them. In this method, the Lagrangian density of the system is constructed and a suitable trial function for the pulse is assumed. With the Lagrangian density, total Lagrangian of the system is constructed. Variation of the Lagrangian with respect to various free parameters appearing in the trial function gives ordinary differential equations (ODE) for the parameters. Pulse evolution in space and time is studied through these ODEs. In many cases, it is not possible to construct a Lagrangian and one has to look for alternate method. The particle like behaviour of the soliton has lead to the formulation of a very powerful and elegant method known as collective variable (CV) method. The CV method was first proposed by Boesh et al. [13] for Klein Gordon (KG) equation. The CV approach is equivalent to the Lagrangian variational method under bare approximation. KG equation is a real equation, which was modified by Tchofo Dinda et al. [14] to make this method applicable to complex equation, particularly for nonlinear Schrödinger equation (NLSE). Pulse dynamics of the HONLSE is studied by the ODEs obtained through the CV technique for different pulse parameters. Fewo et al. [15, 16] have made extensive use of the CV technique for the complex cubic as well as quintic Ginzberg-Landau equation which is perturbed Schrödinger equation and obtained ODEs for the pulse parameters. Nakkeran et al. [17, 18] have employed the technique to Kerr nonlinear system with different ansatz functions. In the CV approach [15], the nonlinear Schrödinger equation (NLSE) is considered as a field equation representing the soliton propagation in the optical fiber link. The field satisfying the NLSE is broken into two parts, the first part is assumed to represent the soliton field while the second part named residual field being responsible for the dressing of the soliton and any other residual radiation. The soliton field is assumed to depend on a collection of variables which may represent amplitude, frequency, chirp, phase etc. Introduction of a number of collective variables increases the available phase space of the dynamical system which is undesirable. The phase space is restricted to the original number by imposing constraints. Bare approximation is applied under which the residual field is set equal to zero. The equations of constraints result in a matrix equation giving ODE for all the collective variables included in the ansatz function chosen to represent the soliton field. These ODE's can be numerically solved to investigate soliton propagation in the fiber. In this paper, employing the collective variable method we have investigated femto second pulse propagation in optical fibers using RKL model. In Sect. 1, we present the derivation of the CV equation of motion. Section 2 is devoted to numerical investigations. A brief conclusion is incorporated in Sect. 3.

2 Mathematical Formulation for CV's Equations of Motions

The field of soliton $\psi(z,t)$ is assumed to be the sum of two parts 'f' and 'g', with f representing the pulse configuration and g being the residual field. The residual field is responsible for the dressing of the soliton and any other radiation coupled to its motion.

$$\Psi(z,t) = f(z,t) + g(z,t). \tag{2}$$

The soliton field f is chosen to be dependent on N variables, called collective variables X_j of the soliton, which in turn are dependent on the variables z and t, i.e.,

$$\Psi(z,t) = f(X_1, X_2, \ldots, X_N, t) + g(z,t), \tag{3}$$

where Ψ is the exact solution of the dynamical field equation, f and g being parts of it, their sum constitutes the exact solution. The introduction of N collective variables in function f increases the degrees of freedom resulting in the expansion of the available phase space of the system, which is undesirable. Therefore, in order that the system remains in the original phase space constraints are imposed. The constraints are obtained by configuring the function f such that it becomes the best fit for static solution. This is obtained by the expression of the residual free energy (RFE) E, which is given by

$$E = \int_{-\infty}^{+\infty} |g|^2 dt = \int_{-\infty}^{+\infty} |\Psi - f(X_1, X_2, \ldots, X_N)|^2 dt. \tag{4}$$

From the residual free energy, we construct two quantities, C_j and $\dot{C}_j$, where C_j describes the rate of change of RFE with respect to the jth CV and $\dot{C}_j$ describes the rate of change of C_j with the normalized distance.

Particularly,

$$C_j = \frac{dE}{dX_j} = \frac{\partial}{\partial X_j}\int_{-\infty}^{\infty} |g|^2 dt = \int_{-\infty}^{\infty}\left(\frac{\partial g}{\partial X_j} g^* + g\frac{\partial g^*}{\partial X_j}\right) dt = \int_{-\infty}^{+\infty} g\frac{\partial g^*}{\partial X_j} dt + \text{c.c.},$$

where c.c. stands for complex conjugate.

Using $g(z,t) = \Psi(z,t) - f\{X_1(z), X_2(z), \ldots, X_N(z), t\}$ in the above equation, the expression of C_j can be rewritten as

$$C_j = \langle f^*_{X_j} g\rangle + \text{c.c.} \tag{5}$$

Similarly

$$\begin{aligned}\dot{C}_j &= \frac{dC_j}{dz} = \frac{d}{dz}\left(\int_{-\infty}^{+\infty} f^*_{X_j} g\, dt\right) + \text{c.c.}\\ &= \int_{-\infty}^{+\infty} f^*_{X_j} g_z dt + \sum_{k=1}^{N}\int_{-\infty}^{+\infty}\frac{\partial}{\partial X_k}(f^*_{X_j})\frac{\partial X_k}{\partial z} g\, dt + \text{c.c.}\end{aligned}$$

Thus,

$$\dot{C}_j = \langle f^*_{X_j} g_z\rangle + \langle f^*_{X_j X_k} g \dot{X}_k\rangle + \text{c.c.}, \tag{6}$$

where dot over a quantity implies differentiation with respect to z, $\langle\cdots\rangle$ means $\int_{-\infty}^{+\infty}(\cdots)dt$, subscript X_j signifies differentiation w.r.t. X_j. Dirac's procedure is employed to obtain the equations of motion for the collective variables, according to which a quantity which is approximately zero, can not be set equal to zero until its variations w.r.t. all variables is made. Hence, the system is assumed to evolve such that C_j are minimum and the equations of constraints are obtained as

$$C_j \cong 0 \quad \text{and} \quad \dot{C}_j \cong 0. \tag{7}$$

We now substitute (3) into (1) to get the equation of motion of the residual field

$$\begin{aligned} g_z &= i f_{tt} + 2i|f+g|^2 f - c_1 f_{ttt} - c_2(|f+g|^2 f)_t - c_3(|f+g|^4 f)_t + i\alpha|f+g|^4 f \\ &\quad - \sum_{j=1}^{N} \frac{\partial f}{\partial X_j} X_j + i g_{tt} + 2i|f+g|^2 g - c_1 g_{ttt} \\ &\quad - c_2(|f+g|^2 g)_t - c_3(|f+g|^4 g)_t + i\alpha|f+g|^4 g, \end{aligned} \tag{8}$$

where we have used $f_z = \sum_{j=1}^{N} \frac{\partial f}{\partial X_j} \dot{X}_j$. Equation (8) on substitution into (6) gives

$$-\dot{C}_j = \sum_{k=1}^{N} \left[2\Re \int_{-\infty}^{+\infty} f^*_{X_j} f_{X_k} dt - 2\Re \int_{-\infty}^{+\infty} f^*_{X_j X_k} g dt \right] \dot{X}_k + R_j. \tag{9}$$

$$\begin{aligned} R &= -2\Re \int_{-\infty}^{+\infty} i f^*_{X_j} f_{tt} dt - 2\Re \int_{-\infty}^{+\infty} 2i f^*_{X_j} |f+g|^2 f dt + 2\Re \int_{-\infty}^{+\infty} c_1 f^*_{X_j} f_{ttt} dt \\ &\quad + 2\Re \int_{-\infty}^{+\infty} c_2 f^*_{X_j} (|f+g|^2 f)_t dt + 2\Re \int_{-\infty}^{+\infty} c_3 f^*_{X_j} (|f+g|^4 f)_t dt \\ &\quad - 2\Re \int_{-\infty}^{+\infty} i\alpha f^*_{X_j} |f+g|^4 f dt - 2\Re \int_{-\infty}^{+\infty} i f^*_{X_j} g_{tt} dt - 2\Re \int_{-\infty}^{+\infty} 2i f^*_{X_j} |f+g|^2 g dt \\ &\quad + 2\Re \int_{-\infty}^{+\infty} c_1 f^*_{X_j} g_{ttt} dt + 2\Re \int_{-\infty}^{+\infty} c_2 f^*_{X_j} (|f+g|^2 g)_t dt \\ &\quad + 2\Re \int_{-\infty}^{+\infty} c_3 f^*_{X_j} (|f+g|^4 g)_t dt - 2\Re \int_{-\infty}^{+\infty} i\alpha f^*_{X_j} |f+g|^4 g dt. \end{aligned} \tag{10}$$

Equation (9) is equivalent to a matrix equation

$$\dot{\mathbf{C}} = \frac{\partial \mathbf{C}}{\partial \mathbf{X}} \cdot \dot{\mathbf{X}} + \mathbf{R}, \quad \text{where } \dot{\mathbf{C}} = (\dot{C}_1\ \dot{C}_2\ \ldots\ \dot{C}_N)^T, \tag{11}$$

$$-\left(\frac{\partial C}{\partial X}\right) = \begin{pmatrix} \frac{\partial C_1}{\partial X_1} & \frac{\partial C_1}{\partial X_2} & \cdots & \frac{\partial C_1}{\partial X_N} \\ \frac{\partial C_2}{\partial X_1} & \frac{\partial C_2}{\partial X_2} & \cdots & \frac{\partial C_2}{\partial X_N} \\ \vdots & \vdots & & \vdots \\ \frac{\partial C_N}{\partial X_1} & \frac{\partial C_N}{\partial X_2} & \cdots & \frac{\partial C_N}{\partial X_N} \end{pmatrix}, \quad X = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{pmatrix} \quad \text{and} \quad R = \left(\left(\begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_N \end{pmatrix}\right)\right), \tag{12}$$

with

$$\frac{\partial C_j}{\partial X_k} = 2\Re \int_{-\infty}^{+\infty} f^*_{X_j} f_{X_k} dt - 2\Re \int_{-\infty}^{+\infty} f^*_{X_j X_k} g dt \tag{13}$$

and

$$R_k = -2\Re\int_{-\infty}^{+\infty} if_{X_j}^* f_{tt}dt - 2\Re\int_{-\infty}^{+\infty} 2if_{X_j}^*|f+g|^2 f dt + 2\Re\int_{-\infty}^{+\infty} c_1 f_{X_j}^* f_{ttt}dt$$
$$+ 2\Re\int_{-\infty}^{+\infty} c_2 f_{X_j}^*(|f+g|^2 f)_t dt + 2\Re\int_{-\infty}^{+\infty} c_3 f_{X_j}^*(|f+g|^4 f)_t dt$$
$$- 2\Re\int_{-\infty}^{+\infty} i\alpha f_{X_j}^*|f+g|^4 f dt - 2\Re\int_{-\infty}^{+\infty} if_{X_j}^* g_{tt}dt - 2\Re\int_{-\infty}^{+\infty} 2if_{X_j}^*|f+g|^2 g dt$$
$$+ 2\Re\int_{-\infty}^{+\infty} c_1 f_{X_j}^* g_{ttt}dt + 2\Re\int_{-\infty}^{+\infty} c_2 f_{X_j}^*(|f+g|^2 g)_t dt$$
$$+ 2\Re\int_{-\infty}^{+\infty} c_3 f_{X_j}^*(|f+g|^4 g)_t dt - 2\Re\int_{-\infty}^{+\infty} i\alpha f_{X_j}^*|f+g|^4 g dt. \quad (14)$$

Employing the constraints equations, the equation of motion for the collective variables are obtained for a chosen form of the function $f(X_1, X_2, \ldots, X_N, t)$. If a Gaussian ansatz is taken containing six CV, f looks like

$$f(X_1, X_2, \ldots, X_N, t) = X_1 e^{\left[-\frac{(t-X_2)^2}{X_3^2} + i\frac{X_4}{2}(t-X_2)^2 + iX_5(t-X_2) + iX_6\right]}, \quad (15)$$

where X_1 represents the pulse amplitude, X_2 the temporal position, X_3 is related to pulse width, X_4 to chirp, X_5 to frequency and X_6 represents the phase of the pulse. Equations for all the collective variables are obtained under lowest order CV theory also known as 'bare approximation'. When dressing of the soliton and radiation associated with the propagation of the soliton is negligible, the bare approximation can be applied under which the residual field is set equal to zero, $g(z,t) = 0$. $(\frac{\partial C}{\partial X})$ and (R) are obtained as

$$\left(\frac{\partial C}{\partial X}\right) =$$
$$\sqrt{2\pi}\begin{pmatrix} X_3 & 0 & \frac{X_1}{2} & 0 & 0 & 0 \\ 0 & \frac{X_3^3}{4}\left(4\frac{X_1^2}{X_3^4} + X_1^2 X_4^2\right) + X_1^2 X_3 X_5^2 & 0 & -\frac{X_1^2 X_3^3 X_5}{8} & -\frac{X_1^2 X_3^3 X_4}{4} & -X_1^2 X_3 X_5 \\ \frac{X_1}{2} & 0 & \frac{3X_1^2}{4X_3} & 0 & 0 & 0 \\ 0 & -\frac{X_1^2 X_3 X_5}{2} & 0 & \frac{3X_1^2 X_3^5}{64} & 0 & \frac{X_1^2 X_3^3}{8} \\ 0 & -\frac{X_1^2 X_3^3 X_4}{4} & 0 & 0 & \frac{X_1^2 X_3^3}{4} & 0 \\ 0 & -X_1^2 X_3 X_5 & 0 & \frac{X_1^2 X_3^3}{8} & 0 & X_1^2 X_3 \end{pmatrix}. \quad (16)$$

Different elements of R matrix are as follows:

$$R_1 = 0, \quad (17a)$$

$$R_2 = \left[-\left(\frac{3X_1^2 X_3^3 X_4^2 X_5}{4} + \frac{3X_1^2 X_5}{X_3} + X_1^2 X_3 X_5^3\right) + \sqrt{2}X_1^4 X_3 X_5\right.$$
$$+ c_1\left(\frac{3X_1^2}{X_3^3} + \frac{3X_1^2 X_3 X_4^2}{2} + \frac{3X_1^2 X_3^5 X_4^4}{16} + \frac{6X_1^2 X_5^2}{X_3} + X_1^2 X_3 X_5^4 + \frac{3X_1^2 X_3^3 X_4^2 X_5^2}{2}\right)$$

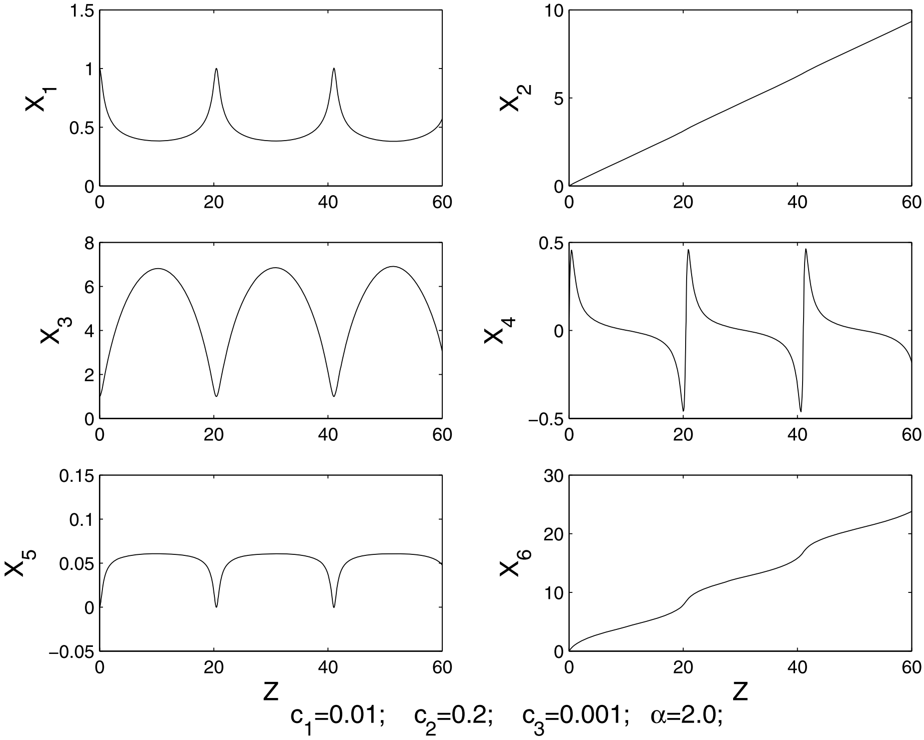

Fig. 1 Variation of pulse width parameters with propagation distance. Values of different system parameters are as follows: $c_1 = 0.01$, $c_2 = 0.2$, $c_3 = 0.001$ and $\alpha = 2.0$

$$- c_2\left(\frac{3X_1^4}{2\sqrt{2}X_3} + \frac{X_1^4X_3^3X_4^2}{8\sqrt{2}} + \frac{X_1^4X_3X_5^2}{\sqrt{2}}\right)$$

$$- c_3\left(\frac{5X_1^6}{3\sqrt{3}X_3} + \frac{X_1^6X_3^3X_4^2}{12\sqrt{3}} + \frac{X_1^6X_3X_5^2}{\sqrt{3}}\right) + \alpha\frac{X_1^6X_3X_5}{\sqrt{3}}\Bigg], \tag{17b}$$

$$R_3 = [-X_1^2X_4 + 3c_1X_1^2X_4X_5], \tag{17c}$$

$$R_4 = \Bigg[-\frac{X_1^2X_3}{8} + \frac{3X_1^2X_3^5X_4^2}{32} + \frac{X_1^2X_3^3X_5^2}{8} - \frac{X_1^4X_3^3}{8\sqrt{2}}$$

$$+ c_1\left(\frac{3X_1^4X_3X_5}{8} - \frac{X_1^2X_3^3X_5^3}{8} - \frac{9X_1^2X_3^5X_4^2X_5}{32}\right)$$

$$+ c_2\frac{X_1^4X_3^3X_5}{16\sqrt{2}} + c_3\frac{X_1^6X_3^3X_5}{24\sqrt{3}} - \alpha\frac{X_1^6X_3^3}{24\sqrt{3}}\Bigg], \tag{17d}$$

$$R_5 = \Bigg[\frac{X_1^2X_3^3X_4X_5}{2} + c_1\left(-\frac{3X_1^2X_3^5X_4^3}{16} - \frac{3X_1^2X_3X_4}{4} - \frac{3X_1^2X_3^3X_4X_5^2}{4}\right)$$

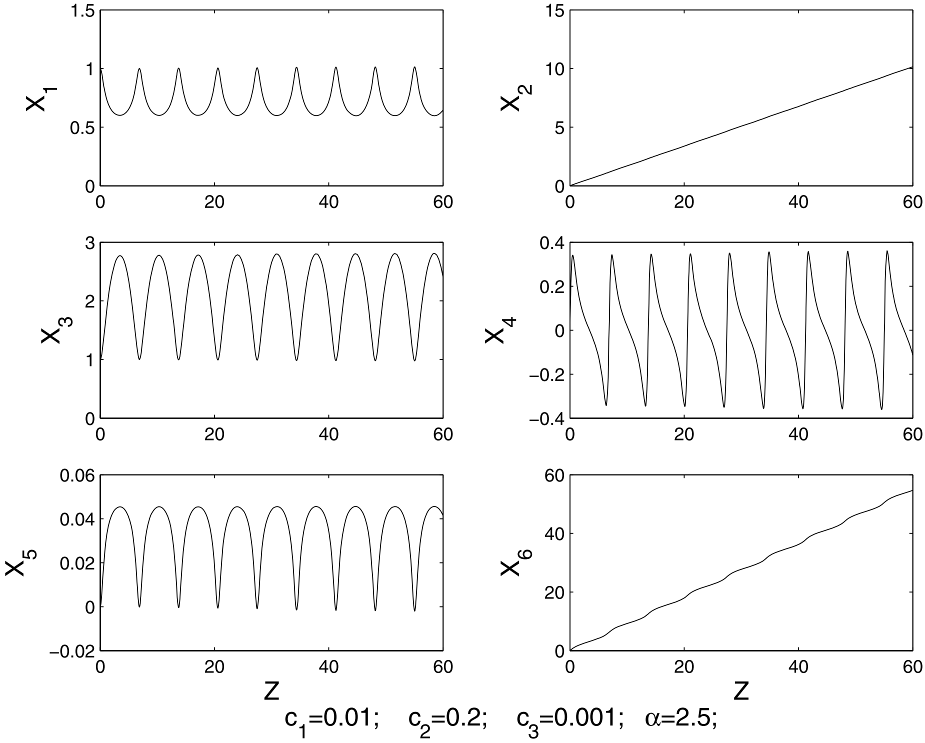

Fig. 2 Variation of pulse width parameters with propagation distance. Values of different system parameters are as follows: $c_1 = 0.01$, $c_2 = 0.2$, $c_3 = 0.001$ and $\alpha = 2.5$

$$+ c_2 \frac{X_1^4 X_3^3 X_4}{8\sqrt{2}} + c_3 \frac{X_1^6 X_3^3 X_4}{12\sqrt{3}} \Bigg], \tag{17e}$$

$$\begin{aligned} R_6 = \Bigg[& \frac{X_1^2}{X_3} + \frac{X_1^2 X_3^3 X_4^2}{4} + X_1^2 X_3 X_5^2 - \sqrt{2} X_1^4 X_3 \\ & + c_1 \left(-\frac{3X_1^2 X_5}{X_3} - X_1^2 X_3 X_5^3 - \frac{3X_1^2 X_3^3 X_4^2 X_5}{4} \right) \\ & + c_2 \frac{X_1^4 X_3 X_5}{\sqrt{2}} + c_3 \frac{X_1^6 X_3 X_5}{\sqrt{3}} - \alpha \frac{X_1^6 X_3}{\sqrt{3}} \Bigg]. \end{aligned} \tag{17f}$$

Finally (11), by virtue of (16) and (17) leads to the following nonlinear Dynamical System:

$$\dot{X}_1 = -X_1 X_4 + c_1 3 X_1 X_4 X_5, \tag{18a}$$

$$\dot{X}_2 = 2X_5 - c_1 \left(\frac{3}{X_3^2} + \frac{3X_3^2 X_4^2}{4} + 3X_5^2 \right) + c_2 \frac{3X_1^2}{2\sqrt{2}} + c_3 \frac{5X_1^4}{3\sqrt{3}}, \tag{18b}$$

$$\dot{X}_3 = 2X_3 X_4 - c_1 6 X_3 X_4 X_5, \tag{18c}$$

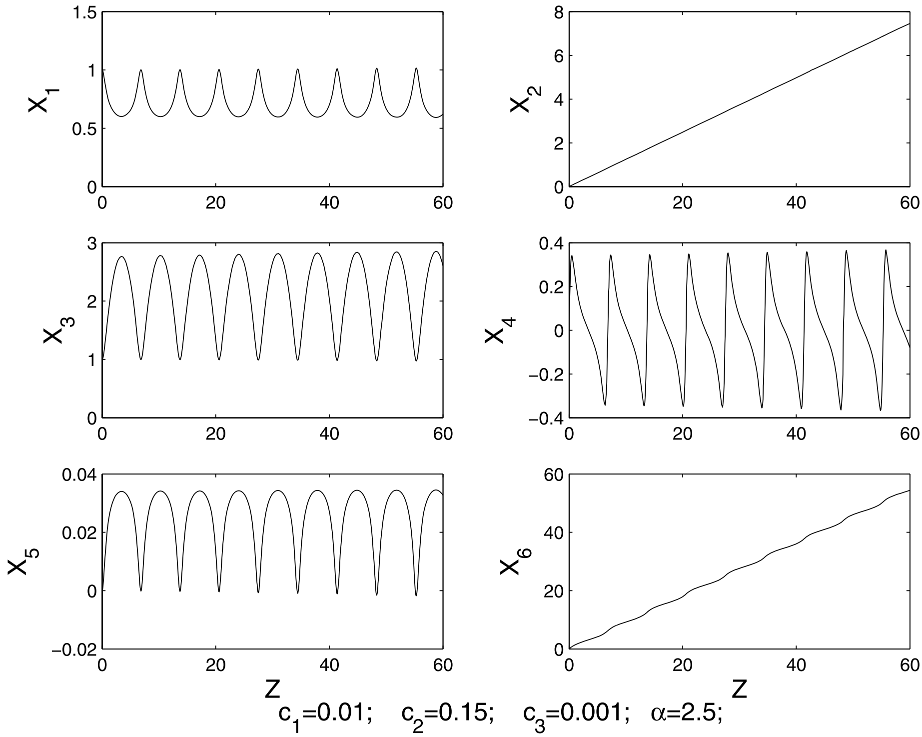

Fig. 3 Variation of pulse width parameters with propagation distance. Values of different system parameters are as follows: $c_1 = 0.01$, $c_2 = 0.15$, $c_3 = 0.001$ and $\alpha = 2.5$

$$\begin{aligned}\dot{X}_4 = {} & \frac{8}{X_3^4} - 2X_4^2 - \frac{2\sqrt{2}X_1^2}{X_3^2} - c_1\left(\frac{24X_5}{X_3^4} - 6X_4^2X_5\right) \\ & + c_2\frac{\sqrt{2}X_1^2X_5}{X_3^2} + c_3\frac{8X_1^4X_5}{3\sqrt{3}X_3^2} - \alpha\frac{8X_1^4}{3\sqrt{3}X_3^2},\end{aligned} \tag{18d}$$

$$\dot{X}_5 = c_2\frac{X_1^2X_4}{\sqrt{2}} + c_3\frac{4X_1^4X_4}{3\sqrt{3}}, \tag{18e}$$

$$\begin{aligned}\dot{X}_6 = {} & X_5^2 - \frac{2}{X_3^2} + \frac{5X_1^2}{2\sqrt{2}} + c_1\left(\frac{3X_5}{X_3^2} - \frac{3X_3^2X_4^2X_5}{4} - 2X_5^2\right) \\ & + c_2\frac{X_1^2X_5}{4\sqrt{2}} + c_3\frac{X_1^4X_5}{3\sqrt{3}} + \alpha\frac{4X_1^4}{3\sqrt{3}}.\end{aligned} \tag{18f}$$

3 Result and Conclusion

To illustrate results of collective variable treatment for ultra short pulse propagation in optical fibers, we have carried out numerical investigations on the evolution of pulse parameters along the propagation direction z. In order to do this we have used standard fourth order

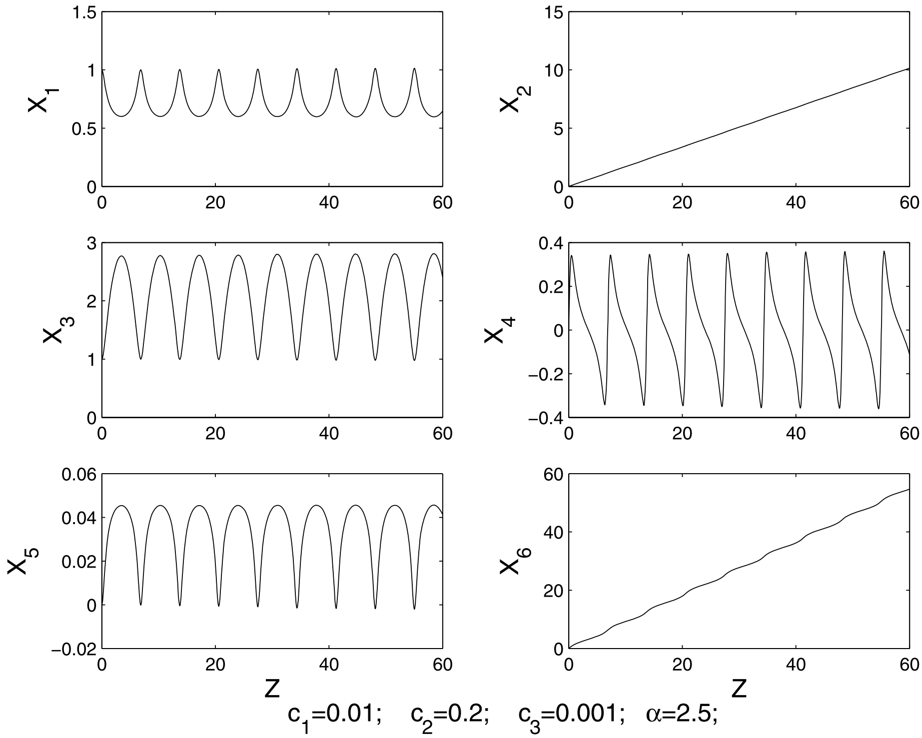

Fig. 4 Variation of pulse width parameters with propagation distance. Values of different system parameters are as follows: $c_1 = 0.01$, $c_2 = 0.2$, $c_3 = 0.001$ and $\alpha = 2.5$

Runge-Kutta method to integrate the system of ordinary differential equations obtained by the CV analysis. Figure 1 represents the dynamics of the pulse with the following set of fixed initial conditions $X_1 = 1$, $X_2 = 0$, $X_3 = 1$, $X_4 = 0$, $X_5 = 0$ and $X_6 = 0$. In Figs. 1 and 2, the value of the quintic coefficient α has been chosen 2.0 and 2.5 respectively. As the pulse propagates, the amplitude (X_1), pulse width (X_3), frequency (X_5) and chirp (X_4) vary periodically. The temporal position of the pulse increases with the increase in the distance of propagation, this may be attributed to intra pulse Raman scattering. With the increase in the value of the quintic nonlinearity, the different pulse parameters such as amplitude, width, chirp and frequency oscillate with higher frequency. The shift of the pulse position remain unaffected with the change in quintic nonlinearity. In order to examine the effect of self steepening on the pulse propagation, we have studied pulse parameters for two different values of c_2, particularly $c_2 = 0.15$ and $c_2 = 0.2$. For these two values of c_2 the pulse dynamics has been depicted in Figs. 3 and 4 respectively. A closer examination of Figs. 3 and 4 shows that the influence of self steepening is largest on the beam center. In conclusion, we have investigated the dynamics of an ultrashort pulse in optical fibers employing CV method.

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